Some Topics of Great Current Interest Zhejiang U, Hangzhou 13 April, 2015 In Heavy Fermion Physics

Piers Coleman: Rutgers Center for Materials Theory, USA





Royal Holloway University of London



Piers Coleman: Rutgers Center for Materials Theory, USA









Quantum Criticality & Strange Metals









Quantum Criticality & Strange Metals



Heavy Fermion Superconductivity









Quantum Criticality & Strange Metals





<u>Heavy Fermion</u> Superconductivity











Quantum Criticality & Strange Metals





Heavy Fermion Superconductivity





Hidden Order





Outline of the Topics

- 1. Introduction: Heavy Fermions and the Kondo Lattice.
- 2. Quantum Criticality and Strange Metals.
- 3. Heavy Fermion Superconductivity
- 4. Topological Kondo Insulators
- 5. Hidden Electron Order



Outline of the Topics

- 1. Introduction: Heavy Fermions and the Kondo Lattice.
- 2. Quantum Criticality and Strange Metals.
- 3. Heavy Fermion Superconductivity
- 4. Topological Kondo Insulators
- 5. Hidden Electron Order

Please ask questions!



Collaborators.

Q. Si
R. Ramazashvili
C. Pepin
Aline Ramires

Rice Toulouse CEA, Saclay Rutgers

Rebecca Flint Premi Chandra Iowa State Rutgers

Andriy NevidomskyyRicAlexei TsvelikBrHai Young KeeU.Natan AndreiRuOnur ErtenRu

Rice Brookhaven NL U. Toronto Rutgers Rutgers

Maxim Dzero Victor Galitski Kai Sun Kent State U. Maryland U. Michigan

Experimentalists:

H. von Lohneysen	Karlsruhe
G. Aeppli	ETH, Zurich
A. Schröder	Kent State
S. Nakatsuji	ISSP
G. Lonzarich	Cambridge
S. Paschen	Vienna
J. Thompson	Los Alamos
J. Allen	U. Michigan
Z. Fisk	UC Irvine
F Steglich	Dresden/Zhejiang





Notes:

"Introduction to Many Body Physics", Ch 8,15-16", PC, CUP to be published (2015).



"Heavy Fermions: electrons at the edge of magnetism." Wiley encyclopedia of magnetism. PC. cond-mat/0612006.
"I2CAM-FAPERJ Lectures on Heavy Fermion Physics", (X=I, II, III) http://physics.rutgers.edu/~coleman/talks/RIO13_X.pdf

<u>General reading:</u>

A. Hewson, "Kondo effect to heavy fermions", CUP, (1993). "The Theory of Quantum Liquids", Nozieres and Pines (Perseus 1999).



Lecture 1 Introduction to Heavy Fermions and the Kondo Lattice.

- 1. Fruit Fly of the 21st Century.
- 2. Electrons on the Brink of Localization.
- 3. Cartoon introduction to Heavy Fermions.
- 4. Anderson & Kondo models
- 5. Doniach Hypothesis.











Fruit-Fly of the 21st C



Heavy Electron Physics PucoGas: 20 K Superconductor Fruit-Fly



r)	ľ	Y	1

Fruit-Fly



Heavy Electron Physics

PuCoGa₅: 20 K Superconductor

the 21st

C



nm L	QUANTUM EMERGENCE	µm
	Ψ	

Heavy Electron Physics

Fruit-Fly

PuCoGa₅: 20 K Superconductor

the 21st



nm	QUANTUM EMERGENCE	µm
	Ψ	

Heavy Electron Physics



ne 2151

Fruit-Fly



m L	QUANTUM EMERGENCE	µm
	T	



SCREECE EREERE

Fruit-FIV

PuCoGa₅ : 20 K Superconductor

the 2151

Electrons on the brink of localization









Smith and Kmetko (1983)









Diversity of new ground-states on the brink of localization.



Cu

Ag

Au

Tc=0.2 -18.5 K

Diversity of new ground-states on the brink of localization.

f-electron systems: 4f Ce, Yb systems 5f U, Np, Pu systems.



Diversity of new ground-states on the brink of localization.

f-electron systems: 4f Ce, Yb systems 5f U, Np, Pu systems.

d-electron systems: e.g Pnictides, Cuprate SC.



HF 115s T_c=0.2 -18.5 K Iron based sc $T_c= 6 - 53 ++ ? K$ Cuprates T_c=11-92K

A new era of mysteries

Cartoon Introduction to Heavy Fermions

Heavy Fermions + Kondo



Spin (4f,5f): basic fabric of heavy electron physics.

Heavy Fermions + Kondo



Spin (4f,5f): basic fabric of heavy electron physics.

Heavy Fermions + Kondo



Spin (4f,5f): basic fabric of heavy electron physics.


Spin (4f,5f): basic fabric of heavy electron physics.



Spin (4f,5f): basic fabric of heavy electron physics.

Scales to Strong Coupling

 $H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + J \vec{S} \cdot \vec{\sigma}(0)$ $\mathbf{k}\sigma$ J. Kondo, 1962

Electron sea

2j+1

χ $\chi \sim 1/T$ Curie T

Spin (4f,5f): basic fabric of heavy electron physics.

Scales to Strong Coupling

 $H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \vec{S} \cdot \vec{\sigma}(0)$ J. Kondo, 1962











Electron sea



Spin screened by conduction electrons: <u>entangled</u>

$$\uparrow \downarrow - \downarrow \uparrow$$



Electron sea



Spin screened by conduction electrons: <u>entangled</u>

$$\uparrow \downarrow - \downarrow \uparrow$$

$$S(T) = \int_0^T \frac{C_V}{T'} dT'$$

Spin entanglement entropy



Electron sea



Spin screened by conduction electrons: <u>entangled</u>

$$\uparrow \downarrow - \downarrow \uparrow$$

$$S(T) = \int_0^T \frac{C_V}{T'} dT'$$

Spin entanglement entropy



Heavy Fermion Primer





"Kondo Lattice"



"Kondo Lattice"

Entangled spins and electrons

→ <u>Heavy Fermion Metals</u>



Entangled spins and electrons → <u>Heavy Fermion Metals</u>





"Kondo Lattice"

Entangled spins and electrons → <u>Heavy Fermion Metals</u>



Coherent Heavy Fermions





Coherent Heavy Fermions



→ <u>New kinds of insulator</u>

Kondo Insulators





→ <u>New kinds of insulator</u>

Topological Kondo Insulators



10¹

10[°]

10⁻¹

(b)

















Quantum Criticality

The Anderson and Kondo Models







$$f^{\dagger}_{\sigma} = \int_{\mathbf{r}} \Psi_f(\mathbf{r}) \hat{\psi}^{\dagger}_{\sigma}(r),$$







$H_{atomic} = E_f n_f + U n_{f\uparrow} n_{f\downarrow}.$









 $\begin{bmatrix} E_f & < & 0 \\ U + E_f & > & 0 \end{bmatrix} \Rightarrow \begin{cases} E_f + U/2 & < & U/2 \\ U/2 & > & -(E_f + U/2) \end{cases}$










Valence Fluctuations





Virtual Valence fluctuations in the singlet channel, induced by hybridization

$$\begin{array}{ll} e_{\uparrow}^{-} + f_{\downarrow}^{1} \leftrightarrow f^{2} \leftrightarrow e_{\downarrow}^{-} + f_{\uparrow}^{1} & \Delta E_{I} \sim U + E_{f} \\ h_{\uparrow}^{+} + f_{\downarrow}^{1} \leftrightarrow f^{0} \leftrightarrow h_{\downarrow}^{+} + f_{\uparrow}^{1} & \Delta E_{II} \sim -E_{f} \end{array}$$



Virtual Valence fluctuations in the singlet channel, induced by hybridization

$$\begin{array}{ll} e_{\uparrow}^{-}+f_{\downarrow}^{1} \leftrightarrow f^{2} \leftrightarrow e_{\downarrow}^{-}+f_{\uparrow}^{1} & \Delta E_{I} \sim U+E_{f} \\ h_{\uparrow}^{+}+f_{\downarrow}^{1} \leftrightarrow f^{0} \leftrightarrow h_{\downarrow}^{+}+f_{\uparrow}^{1} & \Delta E_{II} \sim -E_{f} \end{array}$$

From second order perturbation theory, the energy of c-f singlets reduces by an amount 2J, where

$$J = V^2 \left[\frac{1}{\Delta E_1} + \frac{1}{\Delta E_2} \right]$$



Virtual Valence fluctuations in the singlet channel, induced by hybridization

$$\begin{array}{ll} e_{\uparrow}^{-} + f_{\downarrow}^{1} \leftrightarrow f^{2} \leftrightarrow e_{\downarrow}^{-} + f_{\uparrow}^{1} & \Delta E_{I} \sim U + E_{f} \\ h_{\uparrow}^{+} + f_{\downarrow}^{1} \leftrightarrow f^{0} \leftrightarrow h_{\downarrow}^{+} + f_{\uparrow}^{1} & \Delta E_{II} \sim -E_{f} \end{array}$$

From second order perturbation theory, the energy of c-f singlets reduces by an amount 2J, where

$$J = V^2 \left[\frac{1}{\Delta E_1} + \frac{1}{\Delta E_2} \right]$$



$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{J}{N} \sum_{j} \vec{S}_{j} \cdot c_{\mathbf{k}\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}'\beta} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{R}_{j}}$$

Conduction sea
$$E_{f} \longrightarrow f^{1}$$
$$H_{K} = -2JP_{S=0} = -2J \left[\frac{1}{4} - \frac{1}{2} \vec{\sigma}_{c}(0) \cdot \vec{S}_{f} \right] \rightarrow J \vec{\sigma}_{c}(0) \cdot \vec{S}_{f}$$

Antiferromagnetic Kondo interaction

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{J}{N} \sum_{j} \vec{S}_{j} \cdot c_{\mathbf{k}\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}'\beta} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{R}_{j}}$$
Note: can also write Kondo interaction in the "Coqblin Schrieffer" form
$$H_{K} = -J \sum_{j,\alpha,\beta} (c_{j\alpha}^{\dagger} f_{j\alpha})(f_{j\beta}^{\dagger} c_{j\beta})$$

$$E_{f} \longrightarrow f^{1}$$

$$H_{K} = -2JP_{S=0} = -2J \left[\frac{1}{4} - \frac{1}{2} \vec{\sigma}_{c}(0) \cdot \vec{S}_{f} \right] \rightarrow J \vec{\sigma}_{c}(0) \cdot \vec{S}_{f}$$
Antiferromagnetic Kondo interaction

Doniach Hypothesis



 $\left| H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum (\Psi^{\dagger}_{j} \vec{\sigma} \Psi_{j}) \cdot \vec{S}_{j} \right|$





 $\left| H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum (\psi^{\dagger}_{j} \vec{\sigma} \psi_{j}) \cdot \vec{S}_{j} \right|$



 $T_{RKKY} > T_K$

 $\left| H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum (\psi^{\dagger}_{j} \vec{\sigma} \psi_{j}) \cdot \vec{S}_{j} \right|$

 $T_K \sim D \exp\left[-\frac{1}{2J\rho}\right]$



 $\left| H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum (\psi^{\dagger}_{j} \vec{\sigma} \psi_{j}) \cdot \vec{S}_{j} \right|$





 $T_{RKKY} > T_K$

 $H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum (\psi^{\dagger}_{j} \vec{\sigma} \psi_{j}) \cdot \vec{S}_{j}$

 $T_K \sim D \exp\left[-\frac{1}{2J\rho}\right]$

Kondo Lattice Model (Kasuya, 1951)

 $T_{RKKY} < T_K$

 $T_{RKKY} \sim J^2 \rho$



 $H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum (\Psi^{\dagger}_{j} \vec{\sigma} \Psi_{j}) \cdot \vec{S}_{j}$ Kondo Lattice Model (Kasuya, 1951)

 $T_{RKKY} \sim J^2 \rho$

 $T_K \sim D \exp\left[-\frac{1}{2J\rho}\right]$

 $T_{RKKY} < T_K$

Large Fermi surface of composite Fermions







 $T_{RKKY} > T_K$

The main result ... is that there should be a secondorder transition at zero temperature, as the exchange is varied, between an antiferromagnetic ground state for weak J and a Kondo-like state in which the local moments are quenched.

 $T_K \sim D \exp \left| -\frac{1}{2J\rho} \right|$

 $T_{RKKY} \sim J^2 \rho$

 $T_{RKKY} < T_K$

Large Fermi surface of composite Fermions







 $T_{RKKY} > T_K$

The main result ... is that there should be a secondorder transition at zero temperature, as the exchange is varied, between an antiferromagnetic ground state for weak J and a Kondo-like state in which the local moments are quenched.

 $T_K \sim D \exp \left| -\frac{1}{2J\rho} \right|$

 $T_{RKKY} \sim J^2 \rho$

 $T_{RKKY} < T_K$

Large Fermi surface of composite Fermions





THE KONDO LATTICE

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{J}{\mathcal{N}} \sum_{j} \vec{S}_{j} \cdot c_{\mathbf{k}\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}'\beta} e^{i(\mathbf{k}'-\mathbf{k}) \cdot \mathbf{R}_{j}}$$
T. Kasuya (1951)



"Kondo Lattice"

$$H = J \sum_{\sigma} \vec{\sigma}(j) \cdot \vec{S}_j - t \sum_{(i,j)} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c})$$

$$H = J \sum_{\sigma} \vec{\sigma}(j) \cdot \vec{S}_j - t \sum_{(i,j)} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c})$$



 $n_e = n_{
m spins}$ Kondo insulator

$$H = J \sum_{\sigma} \vec{\sigma}(j) \cdot \vec{S}_j - t \sum_{(i,j)} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c})$$



 $n_e = n_{
m spins}$ Kondo insulator



Electron doping

$$H = J \sum_{\sigma} \vec{\sigma}(j) \cdot \vec{S}_j - t \sum_{(i,j)} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c})$$



 $n_e = n_{
m spins}$ Kondo insulator



Electron doping

$$H = J \sum_{\sigma} \vec{\sigma}(j) \cdot \vec{S}_j - t \sum_{(i,j)} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c})$$



 $n_e = n_{
m spins}$ Kondo insulator



Electron doping Mobile "Heavy Electrons"

$$H = J \sum_{\sigma} \vec{\sigma}(j) \cdot \vec{S}_j - t \sum_{(i,j)} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c})$$



 $n_e = n_{
m spins}$ Kondo insulator





$$H = J \sum_{\sigma} \vec{\sigma}(j) \cdot \vec{S}_j - t \sum_{(i,j)} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c})$$

 $n_e = n_{\rm spins}$

Kondo insulator



Hole doping: mobile heavy holes $n_e = n_{\rm spins} - \delta$



$$H = J \sum_{\sigma} \vec{\sigma}(j) \cdot \vec{S}_j - t \sum_{(i,j)} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c})$$



Hole doping: mobile heavy holes $n_e = n_{\rm spins} - \delta$

 $n_e = n_{
m spins}$ Kondo insulator





$$H = J \sum_{\sigma} \vec{\sigma}(j) \cdot \vec{S}_j - t \sum_{(i,j)} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c})$$



Hole doping: mobile heavy holes $n_e = n_{\rm spins} - \delta$

 $n_e = n_{
m spins}$ Kondo insulator





$$H = J \sum_{\sigma} \vec{\sigma}(j) \cdot \vec{S}_j - t \sum_{(i,j)} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c})$$



 $n_e = n_{
m spins}$ Kondo insulator

$$2\left(\frac{v_{\rm FS}}{(2\pi)^D}\right) = 2 - \delta = n_{\rm spins} + n_e$$

FS sum rule counts spins as charged qp.



Hole doping: mobile heavy holes $n_e = n_{\rm spins} - \delta$



Large Fermi surface and the charge of the f-electron



Summary of Part 1.

- Heavy Fermions: "Fruit fly" of condensed matter physics.
- Interesting physics at the border of magnetism.
- Local moments can entangle with electrons to produce new states of matter emergent physics that we have only just begun to explore and understand.

Lecture 2 Quantum Criticality and Strange Metals

- 1. Lev Landau meets Ken Wilson
- 2. Quantum criticality as a driver of new states of matter.
- 3. Quantum critical heavy fermion systems
- 4. Frustration and Strange Metals



Lev Landau vs Ken Wilson:

Criticality as a driver of new States of Matter



Landau: interactions can be turned on adiabatically, preserving the excitation spectrum.


"Quasiparticle" interactions in







"Quasiparticle" Interactions adiabatically e^{-} |qp| $\frac{m^*}{m} = \frac{N(0)^*}{N(0)} = 1 + \frac{F_1^s}{3}$

Landau, JETP 3, 920 (1957)







Landau, JETP 3, 920 (1957)

$$E_{\mathbf{p}} = \frac{p^2}{2m^*}, \qquad N^*(0) = \frac{m^* p_F}{\pi^2 \hbar^3}$$

10-1







 $\chi(0)$ (emu/mole f atom)



Cu



 $m^*/m_e \sim 1000$











20. Moscow, 1956. Freeman Dyson (front, left),

chuk and Lev Landau.





Long range order

Fermi Liquid

What happens when the interaction becomes too large?

Peierls/Mott 1939

 X_{c}

Wigner/ Landau 1934/36



"Electrons order"



"Electrons localize"





What happens when the interaction becomes too large?

Wigner/ Landau 1934/36



"Electrons order"

Peierls/Mott 1939



"Electrons localize"

Anderson 1961



"Moments form"



What happens when the interaction becomes too large?

Wigner/ Landau 1934/36



"Electrons order"

Peierls/Mott 1939



"Electrons localize"

Anderson 1961



"Moments form"



What happens when the interaction becomes too large?

Wigner/ Landau 1934/36



"Electrons order"

Peierls/Mott 1939



"Electrons localize"

Anderson 1961



"Moments form"

Kenneth Wilson 1936-2013



New Fixed Points



Mott, 1973 Doniach 1976

Wilson 1975



New Fixed Points



Mott, 1973 Doniach 1976

Wilson 1975



New Fixed Points

Quantum Criticality







H. Von Lohneyson (1996)





H. Von Lohneyson (1996)











- ∝

ε

2.0

1.5

1.0

 $B - B_c$









Moriya

Schrieffer

Hertz

- •Moriya, Doniach, Schrieffer (60s) •Hertz (76)
- •Millis (93)

$$d_{eff} = d + z$$













Doniach

Schrieffer Hertz

- •Moriya, Doniach, Schrieffer (60s) •Hertz (76)
- •Millis (93)

 $d_{eff} = d + z$



 $\chi^{-1}(q,\omega) \propto (\xi^{-2} + (q-Q)^2 - i\omega/\Gamma)$











Moriya

Schrieffer

Hertz

- Moriya, Doniach, Schrieffer (60s)Hertz (76)
- •Millis (93)

 $d_{eff} = d + z$



 $\chi^{-1}(q,\omega) \propto (\xi^{-2} + (q-Q)^2 - i\omega/\Gamma)$ $\tau^{-1} \propto \xi^{-2}$











Moriya

Schrieffer

Hertz

- •Moriya, Doniach, Schrieffer (60s) •Hertz (76)
- •Millis (93)

 $d_{eff} = d + z$



$$\chi^{-1}(q, \omega) \propto (\xi^{-2} + (q - Q)^2 - i\omega / \Gamma)$$
$$\tau^{-1} \propto \xi^{-2}$$
Time counts as z = 2 scaling dimensions


Standard Model: Quantum SDW?







Hertz





- •Moriya, Doniach, Schrieffer (60s) •Hertz (76)
- •Millis (93)

 $d_{eff} = d + z$



- If d + z = d + 2 > 4: ϕ^4 terms "irrelevent" Critical modes are Gaussian. T is not the only energy scale.

 $\chi^{-1}(q,\omega) \propto (\xi^{-2} + (q-Q)^2 - i\omega/\Gamma)$



<u>Time counts as z =2 scaling dimensions</u>



$$V_{eff}(\vec{q},\omega) = g^2 \frac{\chi_o}{\left(\vec{q} - \vec{Q}\right)^2 - i\omega / \Gamma_Q}$$

$$V_{eff}(\vec{r},\omega=0) \propto \frac{1}{r}e^{i\vec{Q}.\vec{r}}$$

Singular potential is <u>rapidly modulated</u>: only affects electrons along hot-lines.

 $\varepsilon_{k_F} = \varepsilon_{k_F+Q}$

Predicts:



$$V_{eff}(\vec{q},\omega) = g^2 \frac{\chi_o}{(\vec{q}-\vec{Q})^2 - i\omega/\Gamma_Q}$$

$$V_{eff}(\vec{r},\omega=0) \propto \frac{1}{r} e^{i\vec{Q}.\vec{r}}$$
Singular potential is rapidly modulated:
only affects electrons along hot-lines.

 $\varepsilon_{k_F} = \varepsilon_{k_F+Q}$

Predicts:





Predicts:





Predicts:





Predicts:



$$V_{eff}(\vec{q},\omega) = g^{2} \frac{\chi_{o}}{(\vec{q}-\vec{Q})^{2}-i\omega/\Gamma_{Q}}$$

$$V_{eff}(\vec{r},\omega=0) \propto \frac{1}{r} e^{i\vec{Q},\vec{r}}$$
Singular potential is rapidly modulated:
only affects electrons along hot-lines.

$$\varepsilon_{k_{F}} = \varepsilon_{k_{F}+Q}$$

$$F_{Singular} \sim T\sum_{q} \int d^{3}q \log[\chi^{-1}(q,\omega)]$$

$$\sim T(T^{3/z}) \sim T^{5/2}$$

$$F_{redicts:}$$
Landau's Fermi Liquid Should
Survive at a Quantum Critical Point.

Rutgers Center for Materials Theory



Meigan Aronson



Rutgers Center for Materials Theory



Meigan Aronson

$$\chi''(E) = \frac{1}{E^{1-\alpha}} G(\frac{E}{T})$$







Meigan Aronson



Almut Schroeder

 $\chi''(E) = \frac{1}{E^{1-\alpha}} G(\frac{E}{T})$







Meigan Aronson



Almut Schroeder

 $\chi''(E) = \frac{1}{E^{1-\alpha}} G(\frac{E}{T})$

Physics Below the upper Critical Dimension.





"How do fermions get heavy and die?" PC, Pepin, Si and Ramazashvili, J. Cond Matt., 13}, R723 (2001).

anticipated an abrupt change in FS when a composite heavy electron undergoes a Kondo "breakdown".











S. Paschen et al, Nature 432, 881 (2004)





S. Paschen et al, Nature 432, 881 (2004)



Jump in the Hall constant at a field tuned QCP.











Shimuzu et al (2006)





Shimuzu et al (2006)





Shimuzu et al (2006)



The Quantum Criticality Debate.

• Local quantum criticality

(Si, Ingersent, Smith, Rabello, Nature 2001): Spin is the critical mode, Fluctuations critical in time.

Requires a two dimensional spin fluid

Si, Ingersent





• Local quantum criticality

(Si, Ingersent, Smith, Rabello, Nature 2001): Spin is the critical mode, Fluctuations critical in time.

Requires a two dimensional spin fluid

•Two fluid scenario.



D. Pines Z. Fisk S. Nakatsuji Y. Yang

Nature (2008), PRL (2004)

Si, Ingersent



CeRhIn₅

H = 0

ΡM

p_= 2.5 GPa

SC

3

4

0

 p_c^*

p (GPa)

a)

 AF

1

3

(¥) 2

1

0

0



3

2

T (K)

1

Η

• Local quantum criticality

(Si, Ingersent, Smith, Rabello, Nature 2001): Spin is the critical mode, Fluctuations critical in time.

Η Si, Ingersent 15 CeRhIn₅ b CeRhIn₅ a) p = 2.4 GPa H = 0AF 3 p_= 2.5 GPa ΡM 10 PM (¥) 2 ЕH AF 5 1 SC 0 3 0 3 4 0 2 p (GPa) T (K) Magnetism LM + Kóndo Pure Kondo

Requires a two dimensional spin fluid

•Two fluid scenario.



D. Pines Z. Fisk S. Nakatsuji Y. Yang

Nature (2008), PRL (2004)

Nature (2008), PRL (2004)

• Local quantum criticality

(Si, Ingersent, Smith, Rabello, Nature 2001): Spin is the critical mode, Fluctuations critical in time.



Description of unconventional QCP requires new formalism.

• Local quantum criticality

(Si, Ingersent, Smith, Rabello, Nature 2001): Spin is the critical mode, Fluctuations critical in time.

Magnetism LM + Kóndo

Η Si, Ingersent 15 CeRhIn_e CeRhIn H = 0p = 2.4 GPa AF 3 = 2.5 GPa PM 10 PM (¥) 2 ЕH AF 5 1 SC 0 3 4 0 3 p (GPa) T (K)

Requires a two dimensional spin fluid





D. Pines Z. Fisk S. Nakatsuji Y. Yang

Nature (2008), PRL (2004)

Supersymmetry?

Coleman, Pepin, Tsvelik (1999) Ramires Coleman (2014) Description of unconventional QCP requires new formalism.

Pure Kondo

• Local quantum criticality

•Two fluid scenario.

D. Pines Z. Fisk S. Nakatsuji Y. Yang

(Si, Ingersent, Smith, Rabello, Nature 2001): Spin is the critical mode, Fluctuations critical in time.

Magnetism



Spin = B,FSpin = FSpin = B

Supersymmetry?

Nature (2008), PRL (2004)

Coleman, Pepin, Tsvelik (1999) Ramires Coleman (2014)

Description of unconventional QCP requires new formalism.

• Local quantum criticality

(Si, Ingersent, Smith, Rabello, Nature 2001): Spin is the critical mode, Fluctuations critical in time.

Η Si, Ingersent 15 CeRhIn_e CeRhIn H = 0p = 2.4 GPa AF 3 p = 2.5 GPa PM 10 PM ¥ 2 ЕH AF 5 1 SC 0 3 4 0 3 p (GPa) T (K) LM + Kóndo Pure Kondo

Requires a two dimensional spin fluid

•Two fluid scenario.



D. Pines Z. Fisk S. Nakatsuji Y. Yang

Nature (2008), PRL (2004)

Spin = B

Magnetism

Spin = B,F Spin = F

•Supersymmetry?

Coleman, Pepin, Tsvelik (1999) Ramires Coleman (2014) Description of unconventional QCP requires new formalism.

Strange Metal = Unbroken Susy?

Frustration and strange metals.

Role of Frustration?

M. D. Nunez-Regueiro, C. Lacroix, and B. Canals, Physica C 282, 1885 (1997).

V. Fritsch et al, PRB 89, 054416 (2014)



1. Qu- frustration

Q. Si, Physica B **378–380**, 23 (2006) J. Custers et al, PRL **104**, 186402 (2010) P. Coleman and A. Nevidomskyy, J LTP, 161: 182–202 (2010)

1. Qu- frustration

Q. Si, Physica B **378–380**, 23 (2006) J. Custers et al, PRL **104**, 186402 (2010) P. Coleman and A. Nevidomskyy, J LTP, 161: 182–202 (2010)










M. D. Nunez-Regueiro, C. Lacroix, and B. Canals, Physica C 282, 1885 (1997).

V. Fritsch et al, PRB 89, 054416 (2014)



M. D. Nunez-Regueiro, C. Lacroix, and B. Canals, Physica C 282, 1885 (1997).

(a) С b (b) Ce(1) Ce(3)Ce(2) CePdAl 12 🛟 spin up 😑 spin down

V. Fritsch et al, PRB 89, 054416 (2014)

M. D. Nunez-Regueiro, C. Lacroix, and B. Canals, Physica C 282, 1885 (1997).









Yosuke Matsumoto, S. Nakatsuji et al (2010).





Yosuke Matsumoto, S. Nakatsuji et al (2010).



$$F_{QC} = B^{3/2} f\left(\frac{T}{B}\right)$$
$$B_c = -0.1 \pm 0.1 \text{mT}$$

Yosuke Matsumoto, S. Nakatsuji et al (2010).





Yosuke Matsumoto, S. Nakatsuji et al (2010).





Yosuke Matsumoto, S. Nakatsuji et al (2010).



$$F_{QC} = B^{3/2} f\left(\frac{T}{B}\right)$$
$$B_c = -0.1 \pm 0.1 \text{mT}$$



Yosuke Matsumoto, S. Nakatsuji et al (2010).





FINE-TUNED CRTICALITY?

Yosuke Matsumoto, S. Nakatsuji et al (2010).





Yosuke Matsumoto, S. Nakatsuji et al (2010).



Yosuke Matsumoto, S. Nakatsuji et al (2010).



Yosuke Matsumoto, S. Nakatsuji et al (2010).

LINE OF FIXED POINTS?



Yosuke Matsumoto, S. Nakatsuji et al (2010).

LINE OF FIXED POINTS?



Open Challenges.

- Heavy Fermion QCPs: a new kind of quantum criticality demands a new kind of understanding.
- Needed: Explanation of universality of
- $C/T \sim Log(T_0/T), \rho \sim T^{1+\alpha}$
- Co-existence heavy fermions & LM AFM = Two fluid behavior?
- Role of Frustration?

The Physics of Heavy Fermion Superconductivity

- 1. Magnetism and Superconductivity: a remarkable convergence.
- 2. Glue vs Fabric: Good, Bad and Ugly Heavy Fermion Superconductors.
- 3. The 115 paradox.
- 4. Composite pairing hypothesis.



Magnetism and Superconductivity: A remarkable convergence



After K. Miyake



Bohr-van Leeuwen Theorem (1911,1921)



After K. Miyake



Bohr-van Leeuwen Theorem (1911,1921)



p' = p - eA

$$H[p, A] = H[p', A = 0]$$

1911 Onnes Hg Discovery of SC CLASSICAL PHYSICS IS UNABLE TO ACCOUNT FOR <u>ANY</u> FORM OF MAGNETISM DIA- FERRO- OR PARA- MAGNETISM.

After K. Miyake



1911 Onnes Hg Discovery of SC CLASSICAL PHYSICS IS UNABLE TO ACCOUNT FOR <u>ANY</u> FORM OF MAGNETISM DIA- FERRO- OR PARA- MAGNETISM.

After K. Miyake











(1882-1974)

London 1937

Rigidity of wavefunction -> DIAMAGNETISM







London 1937 Rigidity of wavefunction -> DIAMAGNETISM $|\Psi\rangle = \prod (u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{-\mathbf{k}\downarrow} c^{\dagger}_{\mathbf{k}\uparrow})|0\rangle$



After K. Miyake

Year-



After K. Miyake





London 1937 Rigidity of wavefunction -> DIAMAGNETISM

 $|\Psi\rangle = \prod (u_{\mathbf{k}} + v_{\mathbf{k}}c^{\dagger}_{-\mathbf{k}\downarrow}c^{\dagger}_{\mathbf{k}\uparrow})|0\rangle$





FIG. 3. Ferromagnetic and superconducting transition temperatures of solid solutions of gadolinium in lanthanum.



After K. Miyake



After K. Miyake





After K. Miyake
We tried to detect any possible magnetic ordering below 1K. Instead we found a sharp superconducting transition at 0.97K, which was reduced by about 0.3K only in a field of 60kOe.

Bell Labs, NJ 1973



PHYSICAL REVIEW B

VOLUME 11, NUMBER 1

1 JANUARY 1975

Electronic properties of beryllides of the rare earth and some actinides



PHYSICAL REVIEW B

VOLUME 11, NUMBER 1

1 JANUARY 1975

Electronic properties of beryllides of the rare earth and some actinides



Steglich 1979



PHYSICAL REVIEW B

VOLUME 11, NUMBER 1

1 JANUARY 1975

Electronic properties of beryllides of the rare earth and some actinides

Volume 43, Number 25

PHYSICAL REVIEW LETTERS

17 December 1979

Superconductivity in the Presence of Strong Pauli Paramagnetism: CeCu₂Si₂

F. Steglich Institut für Festkörperphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

and

J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, and W. Franz II. Physikalisches Institut, Universität zu Köln, D-5000 Köln 41, West Germany

and

H. Schäfer Eduard-Zintl-Institut, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany (Received 10 August 1979; revised manuscript received 7 November 1979)



Steglich 1979



Volume 43, Number 25

PHYSICAL REVIEW LETTERS

17 December 1979

Superconductivity in the Presence of Strong Pauli Paramagnetism: CeCu₂Si₂

F. Steglich Institut für Festkörperphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

and

J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, and W. Franz II. Physikalisches Institut, Universität zu Köln, D-5000 Köln 41, West Germany

and

H. Schäfer Eduard-Zintl-Institut, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany (Received 10 August 1979; revised manuscript received 7 November 1979)



Steglich 1979



Since the Debye temperature, Θ , is of the order of 200 K,⁵ we find $T_c < T_F < \Theta$ with $T_c / T_F \simeq T_F / \Theta$ $\simeq 0.05$. This suggests that CeCu₂Si (i) behaves as a "high-temperature superconductor" and (ii) cannot be described by conventional theory of superconductivity which assumes a typical phonon frequency $k_B \Theta / h \ll k_B T_F / h$, the characteristic frequency of the fermions.



PHYSICAL REVIEW B

VOLUME 11, NUMBER 1

1 JANUARY 1975

Electronic properties of beryllides of the rare earth and some actinides







PHYSICAL REVIEW B

VOLUME 11, NUMBER 1

1 JANUARY 1975

Ott

1976

Electronic properties of beryllides of the rare earth and some actinides

E. Bucher, *J. P. Maita, G. W. Hull, R. C. Fulton, and A. S. Cooper Bell Laboratories, Murray Hill, New Jersey 07974 (Received 14 March 1974) Steglich Fisk 1979 1983









PHYSICAL REVIEW B

VOLUME 11, NUMBER 1

1 JANUARY 1975

Ott S

1976

Steglich Fisk 1979 1983

Electronic properties of beryllides of the rare earth and some actinides

Magnetism and Supercond









Ott Stegl 1976 1979

Steglich Fisk 1979 1983





Fabric vs Glue?

Glue Spin fluctuations = pairing bosons



Glue Spin fluctuations = pairing bosons



Eliashberg Approach (cf B. Keimer et al)



Glue Spin fluctuations = pairing bosons





Fabric: spins make the pairs

Glue Spin fluctuations = pairing bosons





a^{e^}

Fabric: spins make the pairs

Anderson: RVB (1987); Coleman Andrei (1989) Emery & Kivelson: composite pairs (1993)





Conduction e⁻

(a)

spins

Glue Spin fluctuations = pairing bosons



$(\pi,0) \rightarrow s^{\pm}$

Fabric: spins make the pairs

Anderson: RVB (1987); Coleman Andrei (1989) Emery & Kivelson: composite pairs (1993)









"Hilbert Space Spectroscopy"

Glue Spin fluctuations = pairing bosons





Fabric: spins make the pairs

Anderson: RVB (1987); Coleman Andrei (1989) Emery & Kivelson: composite pairs (1993)









"Hilbert Space Spectroscopy"

Glue Spin fluctuations = pairing bosons



$(\pi,0) \rightarrow s^{\pm}$

e⁻ Conduction e⁻ (a) f spins

Fabric: spins make the pairs

Anderson: RVB (1987); Coleman Andrei (1989) Emery & Kivelson: composite pairs (1993)







"Hilbert Space Spectroscopy" SPIN Hilbert space BUILDS the pairs.

Glue Spin fluctuations = pairing bosons



$(\pi,0) \rightarrow s^{\pm}$

Fabric: spins make the pairs

Anderson: RVB (1987); Coleman Andrei (1989) Emery & Kivelson: composite pairs (1993)









"Hilbert Space Spectroscopy" SPIN Hilbert space BUILDS the pairs. How?

Magnetic pairing appears ubiquitous



"FM":	Ulr, UGe2,URhGe, UCoGe
"AFM":	CeCu2Si2, UPt3, CeIn3, CePd2Si2, CeRhIn5,
	CeCoIn5,CeRhIn5,UPd2Al3
Strange:	UBe13, PuCoGa5,NpPd5Al2, URu2Si2
Non centrosymmetric:	CePtSi, CeIrSi3, CeRhSi3
"Quadrupolar":	PrOs4Sb2

Heavy Fermion SC: The Good.

Example: UPt₃ T* ~ 100K, T_C = .56K



Stewart, Fisk, Willis and Smith,

Phys. Rev. Lett. 52, 679-682 (1984)

Example: UPt₃ T* ~ 100K, T_C = .56K



Stage one: QP formation

Pauli paramagnetism fully developed by 30K~50 $\rm T_{\rm C}$

Stewart, Fisk, Willis and Smith,

Phys. Rev. Lett. 52, 679-682 (1984)

Example: UPt₃ T* ~ 100K, T_C = .56K



Stewart, Fisk, Willis and Smith, Phys. Rev. Lett. 52, 679–682 (1984)

Stage one: QP formation

Pauli paramagnetism fully developed by 30K~50 T_C



Example: UPt₃ T* ~ 100K, T_C = .56K



Stewart, Fisk, Willis and Smith, Phys. Rev. Lett. 52, 679–682 (1984)

Stage one: QP formation

Pauli paramagnetism fully developed by 30K~50 T_C





Frings *et al.* J. Magn. Magn. Mater. **31**, 240(1983) Brison *et al.* J. Low Temp. Phys. **95**, 145(1994)



Frings *et al.* J. Magn. Magn. Mater. **31**, 240(1983) Brison *et al.* J. Low Temp. Phys. **95**, 145(1994)



Joynt and Taillefer, RMP 2002.





200

T(K)

300

100

χ (10⁻⁹m³/molU)

0

0

UPt₃: Classic HFSC

Joynt and Taillefer, RMP 2002.





Joynt and Taillefer, RMP 2002.





0

<i>F</i> (MG)		m*/me	
Expt.	Calc.	Expt.	Calc.
<i>a</i> axis	(<i>ГK</i>)		
5.4(3)	10.4	25(3)	2.2
6.0(4)	5.2	• • •	1.0
7.3(3)	8.2	40(7)	2.0
14.0(3)	9.1	50(8)	1.9
21.0(3)	24.0	60(8)	4.6
58.5(5)	52.8	90(15)	5.3
b axis	(<i>ГМ</i>)		
4.1(2)		15(5)	
12.3(2)		30(3)	
15.5(2)		35(7)	
18.7(3)		40(8)	
21.9(4)			
25.1(5)		(50)	
	F (M) Expt. <i>a</i> axis 5.4(3) 6.0(4) 7.3(3) 14.0(3) 21.0(3) 58.5(5) <i>b</i> axis 4.1(2) 12.3(2) 15.5(2) 18.7(3) 21.9(4) 25.1(5)	$F(MG)$ Expt.Calc. $a axis (\Gamma K)$ $5.4(3)$ 10.4 $6.0(4)$ 5.2 $7.3(3)$ 8.2 $14.0(3)$ 9.1 $21.0(3)$ 24.0 $58.5(5)$ 52.8 $b axis (\Gamma M)$ $4.1(2)$ $12.3(2)$ $15.5(2)$ $18.7(3)$ $21.9(4)$ $25.1(5)$	F (MG) $m^*/$ Expt.Calc.Expt. a axis (ΓK) $5.4(3)$ 10.4 $25(3)$ $5.4(3)$ 10.4 $25(3)$ $6.0(4)$ 5.2 \cdots $7.3(3)$ 8.2 $40(7)$ $14.0(3)$ 9.1 $50(8)$ $21.0(3)$ 24.0 $60(8)$ $58.5(5)$ 52.8 $90(15)$ b axis (ΓM) $15(5)$ $12.3(2)$ $30(3)$ $15.5(2)$ $35(7)$ $18.7(3)$ $40(8)$ $21.9(4)$ \cdots $25.1(5)$ (50)





Joynt and Taillefer, RMP 2002.



	F (MG)		m^*/m_e	
Branch:FS orbit	Expt.	Calc.	Expt.	Calc.
	a axis	(<i>ГK</i>)		
$\alpha:ML4$	5.4(3)	10.4	25(3)	2.2
β:L4	6.0(4)	5.2		1.0
γ:Γ1	7.3(3)	8.2	40(7)	2.0
δ:Α5	14.0(3)	9.1	50(8)	1.9
<i>ϵ</i> :Γ2	21.0(3)	24.0	60(8)	4.6
ω:Γ3	58.5(5)	52.8	90(15)	5.3
	b axis	(<i>ГМ</i>)		
α :ML4	4.1(2)		15(5)	
δ:A5	12.3(2)		30(3)	
$\theta:A4,5$	15.5(2)		35(7)	
<i>¢</i> : <i>A</i> 4,5	18.7(3)		40(8)	
ψ:A4,5	21.9(4)			
λ: <i>A</i> 4	25.1(5)		(50)	







Joynt and Taillefer, RMP 2002.





Joynt and Taillefer, RMP 2002.



79


UPt₃: Classic HFSC

Joynt and Taillefer, RMP 2002.



80

Magnetic Order and Fluctuations in Superconducting UPt₃

G. Aeppli and E. Bucher AT&T Bell Laboratories, Murray Hill, New Jersey 07974

C. Broholm and J. K. Kjems

Physics Department, Risø National Laboratory, Roskilde DK-4000, Denmark

and

J. Baumann and J. Hufnagl University of Konstanz, D-7750 Konstanz, Federal Republic of Germany (Received 24 September 1987)





AFM -> d-wave pairing.



AFM -> d-wave pairing.



Conventional Heavy Fermion SC:

superconductivity



How do we get from here to heavy Cooper pairs?

"Conventional" heavy fermion superconductivity



How do we get from here to heavy Cooper pairs?

1. The local moments quench [via the Kondo effect], forming heavy quasiparticles

"Conventional" heavy fermion superconductivity



How do we get from here to heavy Cooper pairs?

- **1.** The local moments quench [via the Kondo effect], forming heavy quasiparticles
- 2. The heavy quasiparticles pair [via residual spin fluctuations]

These two stages are well separated.

Heavy Fermion SC: Bad and Ugly





UBe₁₃



Local Moments



Local Moments





Local Moments





Ott et al, (1983)





· ·























SC

3



The 115 Family. Magnetism appears ubiquitous.



CeXIn₅

Sarrao and Thompson JPSJ (2007)

The 115 Family. Magnetism appears ubiquitous.





Yet...

Sarrao and Thompson JPSJ (2007)

Magnetic pairing appears ubiquitous

But...

- Two domes in CeMIn₅
- Superconductivity without magnetism (PuMIn₅, PuMGa₅, NpPd₅Al₂)
- Extreme robustness to disorder
- Many Ce superconductors, one (weak) Yb superconductor

Are there other possible mechanisms?



Sarrao and Thompson JPSJ (2007)











115 Mystery.

The Mystery of NpPd₅Al₂



The Mystery of NpPd₅Al₂







The Mystery of NpPd₅Al₂


The Mystery of NpPd₅Al₂



4.5K Heavy Fermion S.C NpAl₂Pd₅ Aoki et al 2007

The Mystery of NpPd₅Al₂



How does the spin form the condensate?

4.5K Heavy Fermion S.C NpAl₂Pd₅ Aoki et al 2007







D. Aoki et al., J. Phys. Soc. Jpn. **76** (2007) 063701.





D. Aoki et al., J. Phys. Soc. Jpn. **76** (2007) 063701.





D. Aoki et al., J. Phys. Soc. Jpn. 76 (2007) 063701.





Signals a release of the local moment from the condensate.



D. Aoki et al., J. Phys. Soc. Jpn. 76 (2007) 063701.



Signals a release of the local moment from the condensate.



D. Aoki et al., J. Phys. Soc. Jpn. 76 (2007) 063701.











11.2

8

11.8

12

н

10



Paradox:

How can a neutral magnetic moments form a charged superconducting condensate?

T. Tayama et al., RPB 65, 180504R (2002)

6

H (10kOe)

-1

0

2

Δ





Paradox:

How can a neutral magnetic moments form a charged superconducting condensate?

 $\prod_{\bigotimes j} \{ = \begin{array}{c} & & & \\ & & & \\ & & \\ & & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & & \\ & & \\ & & & \\ & & & \\$

Composite pairing Hypothesis.





















$$\Psi^{\dagger} = c_{1|}^{\dagger} c_{2|}^{\dagger} S_{-}$$



Temperature (K)

 $\Psi^{\dagger} = c_{1 \perp}^{\dagger} c_{2 \perp}^{\dagger} S_{+}$



Abrahams, Balatsky, Scalapino, Schrieffer 1995

A solvable model of composite pairing.

PC, Tsvelik, Kee, Andrei PRB 60, 3605 (1999).
Flint, Dzero, PC, Nature Physics 4, 643 (2008).
Flint, PC, PRL, 105, 246404 (2010).
Flint, Nevidomskyy, PC, PRB 84, 064514 (2011).

$$H = \sum_{k} \epsilon_{k} c_{k}^{\dagger} c_{k} + J_{1} \sum_{j} \psi_{1j\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1k\beta} \cdot \vec{S}_{j}$$





 $|\Gamma_7^+\rangle$

Wannier functions at site j:

$$\psi_{\Gamma j}^{\dagger} = \sum_{k} \Phi_{\Gamma k} \mathrm{e}^{i \vec{k} \cdot \vec{R}_{j}} c_{k}$$

$$H = \sum_{k} \epsilon_{k} c_{k}^{\dagger} c_{k} + J_{1} \sum_{j} \psi_{1j\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1k\beta} \cdot \vec{S}_{j}$$





Wannier functions at site j:

$$\psi_{\Gamma j}^{\dagger} = \sum_{k} \Phi_{\Gamma k} \mathrm{e}^{i\vec{k}\cdot\vec{R}_{j}} c_{k}$$







$$H = \sum_{k} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_{1} \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_{2} \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

cf Cox, Pang, Jarell (96) PC, Kee, Andrei, Tsvelik (98) Single FS, two channels. $\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma \mathbf{k}} \ c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_{j}}$ Impurity: quantum critical point for $J_1 = J_2$



$$H = \sum_{k} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_{1} \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_{2} \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

cf Cox, Pang, Jarell (96) PC, Kee, Andrei, Tsvelik (98) Single FS, two channels.

$$\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma \mathbf{k}} \ c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_{j}}$$

Impurity: quantum critical point for $J_1 = J_2$

Nozieres and Blandin 1980



$$H = \sum_{k} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_{1} \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_{2} \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

cf Cox, Pang, Jarell (96) PC, Kee, Andrei, Tsvelik (98) Single FS, two channels. $\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma \mathbf{k}} \quad \underline{c_{\mathbf{k}}} e^{i\mathbf{k} \cdot \mathbf{x}_{j}}$ Impurity: quantum critical point for $J_1 = J_2$

Nozieres and Blandin 1980



$$H = \sum_{k} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_{1} \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_{2} \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

cf Cox, Pang, Jarell (96) PC, Kee, Andrei, Tsvelik (98) Single FS, two channels.

$$\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma \mathbf{k}} \ c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_{j}}$$

Impurity: quantum critical point for $J_1 = J_2$

Nozieres and Blandin 1980



$$H = \sum_{k} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_{1} \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_{2} \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

cf Cox, Pang, Jarell (96) PC, Kee, Andrei, Tsvelik (98) Single FS, two channels.

$$\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma \mathbf{k}} \ c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_{j}}$$


Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



Composite pair $\Psi_C^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S_+$

Abrahams, Balatsky, Scalapino, Schrieffer 1995

Andrei, Coleman, Kee & Tsvelik PRB (1998) Flint, Dzero, Coleman, Nat. Phys, (2008)

Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



Composite pair: intra-cell boson

 $\Psi_C^{\dagger} = c_{1\downarrow}^{\dagger} c_{2\downarrow}^{\dagger} S_+$

Abrahams, Balatsky, Scalapino, Schrieffer 1995

Andrei, Coleman, Kee & Tsvelik PRB (1998) Flint, Dzero, Coleman, Nat. Phys, (2008)

Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



Extreme Resilience to doping on Ce site.



Lei Shu et al, PRL, (2011)

Composite pair: intra-cell boson $\Psi_C^\dagger = c_{1|}^\dagger c_{2|}^\dagger S_+$

Abrahams, Balatsky, Scalapino, Schrieffer 1995

Andrei, Coleman, Kee & Tsvelik PRB (1998) Flint, Dzero, Coleman , Nat. Phys, (2008)

Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



Composite pair: intra-cell boson

 $\Psi_C^{\dagger} = c_{1\downarrow}^{\dagger} c_{2\downarrow}^{\dagger} S_+$

Abrahams, Balatsky, Scalapino, Schrieffer 1995

Andrei, Coleman, Kee & Tsvelik PRB (1998) Flint, Dzero, Coleman, Nat. Phys, (2008) Extreme Resilience to doping on Ce site.



Lei Shu et al, PRL, (2011) M. Tanatar et al (unpublished) Erten and PC arXiv1402.7361





$$\begin{split} \Psi^{\dagger} &= c_{1\downarrow}^{\dagger} c_{2\downarrow}^{\dagger} S_{+} \\ Q_{zz} \propto \Psi_{C}^{2} \end{split}$$



 $\Delta F \propto -Q_{zz} u_{tet}$



 $\Delta F \propto -Q_{zz} u_{tet}$ $\alpha_2 [T - (T_{c2} + \lambda u_{tet})] \Psi_C^2$



 $\Delta F \propto -Q_{zz} u_{tet}$ $\alpha_2 [T - (T_{c2} + \lambda u_{tet})] \Psi_C^2$

 $\Rightarrow T_c = T_{c2} + \lambda u_{tet}$



$$\Delta F \propto -Q_{zz} u_{tet}$$
$$\alpha_2 [T - (T_{c2} + \lambda u_{tet})] \Psi_C^2$$

 $\Rightarrow T_c = T_{c2} + \lambda u_{tet}$

Strain expected to enhance Tc



 $\Rightarrow T_c = T_{c2} + \lambda u_{tet}$

Strain expected to enhance Tc















$$\frac{1}{\lambda_L^2(T)} = \frac{1}{\lambda_L^2(0)} - \overline{N(\epsilon)}$$





d-wave sc

$$\frac{1}{\lambda_L^2(T)} = \frac{1}{\lambda_L^2(0)} - \overline{N(\epsilon)}$$

Penetration Depth





$$\frac{1}{\lambda_L^2(T)} = \frac{1}{\lambda_L^2(0)} - \overline{N(\epsilon)}$$

Penetration Depth





$$\frac{1}{\lambda_L^2(T)} = \frac{1}{\lambda_L^2(0)} - \overline{N(\epsilon)}$$

Penetration Depth





 $\frac{1}{\lambda_L^2(T)} = \frac{1}{\lambda_L^2(0)} - \overline{N(\epsilon)}$

Penetration Depth





d-wave sc

$$\frac{1}{\lambda_L^2(T)} = \frac{1}{\lambda_L^2(0)} - \overline{N(\epsilon)}$$





$$\frac{1}{\lambda_L^2(T)} = \frac{1}{\lambda_L^2(0)} - \overline{N(\epsilon)}$$





$$\frac{1}{\lambda_L^2(T)} = \frac{1}{\lambda_L^2(0)} - \overline{N(\epsilon)}$$

Penetration Depth







STM: B. Zhou *et al.* Nature Phys. 9, 474 (2013)







STM: B. Zhou *et al.* Nature Phys. 9, 474 (2013)



Thermal cond: K. Izawa *et al.* PRL 87, 057002 (2001)



Torque magnetometry: H. Xiao *et al.* PRB 78, 014510 (2008)





$$\frac{1}{\lambda_L^2(T)} = \frac{1}{\lambda_L^2(0)} - \overline{N(\epsilon)}$$

Penetration Depth





$$\frac{1}{\lambda_L^2(T)} = \frac{1}{\lambda_L^2(0)} - \overline{N(\epsilon)}$$



Penetration Depth



Penetration Depth





But Inconsistent with observed d-wave character??


 $\lambda_L(T) = \lambda_L(0) + aT^n$



Erten, Flint, PC, 2014



$$\overset{\rho_s}{\to} \overset{\gamma_0}{\to} (T)^{(1-2)} \overset{\rho_s}{=} \overset{\rho_0}{\to} \overset{\gamma_0}{\to} \overset{\gamma_0}{\to} \overset{\gamma_0}{\to} \overset{\rho_0}{\to} \overset$$





Erten, Flint, PC, 2014



Erten, Flint, PC, 2014

Open Challenges.

- Convergence of magnetism and superconductivity: require new concepts over and beyond spin fluctuation theory. POTENTIAL FOR DISCOVERY.
- HFSC: how is the spin incorporated into the condensate?
- 115 heavy fermion superconductors suggest a new kind of pairing: composite pairing, robust against disorder on magnetic site.
- Could the same phenomenon occur in d-electron materials, at much higher temperatures?

Topological Kondo insulators and other (hidden) forms of order.

- 1. Heavy Fermions: The Rise of topology.
- 2. Is SmB6 topological?
- 3. Hidden electron Order.
- 4. Another long-lived puzzle: URu2Si2
- 5. Dipoles, multipoles, spinors?



Heavy Fermions

Piers Coleman, Center for Materials Theory, Rutgers, NJ. Royal Holloway, University of London, UK.



UBC Brainstorming Session 14-16 May 2014



Heavy Fermions

Piers Coleman, Center for Materials Theory, Rutgers, NJ. Royal Holloway, University of London, UK.



UBC Brainstorming Session 14-16 May 2014



Heavy Fermions

Piers Coleman, Center for Materials Theory, Rutgers, NJ. Royal Holloway, University of London, UK.



UBC Brainstorming Session 14-16 May 2014

350



Heavy Fermions : The Rise of Topology.

Piers Coleman,

(emu/mole)

×

103

Center for Materials Theory, Rutgers, NJ. Royal Holloway, University of London, UK.

Menth, B let ler & Ge

PRL 22, 295 (* 969)





UBC Brainstorming Session 14-16 May 2014

Sun, V. Galitski, and R. Coleman, Phys. Rev. Lett. **104**, 106408 (2010). Sun, P. Coleman and V. Galitski, Phys. Rev. B **85**, 045130 (2012).







Parent compound of Heavy Fermions

Kondo insulators: Parent compound of Heavy Fermions



Kondo insulators: Parent compound of Heavy Fermions





Kondo insulators: Parent compound of Heavy Fermions



Maple + Wohlleben, PRL, **1971** N. Mott Phil Mag 30,403,**1974**





Parent compound of Heavy Fermions



Maple + Wohlleben, PRL, **1971** N. Mott Phil Mag 30,403,**1974** In SmB₆ and high-pressure SmS a very small gap separates the occupied from unoccupied states, this in our view being due to hybridization of 4f and 5d bands.... it is suggested that this must always occur if the the Kondo temperature is higher than the RKKY interaction.



Parent compound of Heavy Fermions



Maple + Wohlleben, PRL, **1971** N. Mott Phil Mag 30,403,**1974** In SmB₆ and high-pressure SmS a very small gap separates the occupied from unoccupied states, this in our view being due to hybridization of 4f and 5d bands.... it is suggested that this must always occur if the the Kondo temperature is higher than the RKKY interaction.



Kondo Insulator.



Parent compound of Heavy Fermions



Maple + Wohlleben, PRL, **1971** N. Mott Phil Mag 30,403,**1974**



In SmB₆ and high-pressure SmS a very small gap separates the occupied from unoccupied states, this in our view being due to hybridization of 4f and 5d bands.... it is suggested that this must always occur if the the Kondo temperature is higher than the RKKY interaction.

Kondo Insulator.



Parent compound of Heavy Fermions



Maple + Wohlleben, PRL, **1971** N. Mott Phil Mag 30,403,**1974**



Martin and Allen(J. Appl Phys, 49,2078, 1979)

In SmB₆ and high-pressure SmS a very small gap separates the occupied from unoccupied states, this in our view being due to hybridization of 4f and 5d bands.... it is suggested that this must always occur if the the Kondo temperature is higher than the RKKY interaction.

Kondo Insulator.



Parent compound of Heavy Fermions



Maple + Wohlleben, PRL, **1971** N. Mott Phil Mag 30,403,**1974**



Martin and Allen(J. Appl Phys, 49,2078, 1979) Aeppli and Fisk (Comments CMP 16, 155, 1992)

In SmB₆ and high-pressure SmS a very small gap separates the occupied from unoccupied states, this in our view being due to hybridization of 4f and 5d bands.... it is suggested that this must always occur if the the Kondo temperature is higher than the RKKY interaction.

Kondo Insulator.



The Rise of Topology.

 $\vec{a}_{\mathbf{k}\mu\nu} = i\langle u_{\mu,\mathbf{k}} | \vec{\nabla}_{\mathbf{k}} | u_{\nu,\mathbf{k}} \rangle$

Berry Connection ~ Field (texture) in momentum space

Thouless et al Haldane Kane and Mele Bernevig & Zhang Fu and Kane (1982) IQHE
(1987) NO NET FIELD REQD
(2005) Spin orbit: Q. Spin Hall
(2006) Spin orbit: Q. Spin Hall
(2007) 3D Z₂ insulator





 $\vec{a}_{\mathbf{k}\mu\nu} = i \langle u_{\mu,\mathbf{k}} | \vec{\nabla}_{\mathbf{k}} | u_{\nu,\mathbf{k}} \rangle$

Berry Connection ~ Field (texture) in momentum space

Thouless et al
Haldane
Kane and Mele
Bernevig & Zhang
Fu and Kane

(1982) IQHE
(1987) NO NET FIELD REQD
(2005) Spin orbit: Q. Spin Hall
(2006) Spin orbit: Q. Spin Hall
(2007) 3D Z₂ insulator







Are Kondo insulators topological?

Topological Kondo Insulators, Dzero, Sun, Galitski, PC Phys. Rev. Lett. **104**, 106408 (**2010**) Maxim Dzero, Kai Sun, Piers Coleman and Victor Galitski, Phys. Rev. B 85, 045130-045140 (2012).

Tight binding model for SmB₆: T. Takimoto, J. Phys. Soc. Jpn. 80, 123710 (2011). Victor Alexandrov, Maxim Dzero and Piers Coleman Phys. Rev. Lett 111, 226403 (2013). Gutzwiller + Band Theory F. Lu, J. Zhao, H. Weng, Z. Fang and X. Dai, Phys. Rev. Lett. 110, 096401 (2013).







 $\nu = +1$







 $\nu = +1$



 $d^0 f^6$

 $\nu = +1$



 $d^0 f^6$

 $\nu = +1$



 $\begin{array}{ccc} d^0 f^6 & \longrightarrow d^1 f^5 \\ \nu = +1 & \nu = -1 \end{array} \end{array}$



 $\begin{array}{ccc} d^0 f^6 & \longrightarrow d^1 f^5 \\ \nu = +1 & \nu = -1 \end{array}$

THREE DIRAC CONES ON SURFACE. M. Tran et al (2012) F. Lu et al (2013) Alexandrov, Dzero and PC (2013)



D. J. Kim et al, Scientific Reports 3, 3150 (2013)

Wolgast et al, Phys Rev B, 88, 180405 (2013)



Bulk Insulator

Surface Conductivity

Robustness/Sensitivity to potential/magnetic scattering.

(D. J. Kim, Paglione)



D. J. Kim et al, Scientific Reports 3, 3150 (2013)



Wolgast et al, Phys Rev B, 88, 180405 (2013)



d-band crossing at X points

(Zahid Hasan, Xi Dai)

Odd number (3) of Surface FS (ARPES, dHVA, STM). (Zahid Hasan, Xi Dai)



Nature Commnunications 4, (2013).





d-band crossing at X points

(Zahid Hasan, Xi Dai)

(Zahid Hasan, Xi Dai)

Odd number (3) of Surface FS (ARPES, dHVA, STM).



Nature Commnunications 4, (2013).



D43.0001 Nan Xu , X. Shi , P. Biswas , C. Matt , R. Dhaka , Y. Huang , N. Plumb , M. Radovic , J. Dil , E. Pomjakushina , K. Conder , A. Amato , Z. Salman , D. Paul , J. Mesot , Hong Ding , Ming Shi

Spin Resolved ARPES (Courtesy, Hong Ding)



N. Xu et al., Submitted (2013).
Open Challenges.

- Surface Kondo physics
- Why are the surface states so light?
- What aspects of KIs/TKIs are different to their weakly interacting counterparts?

URu₂Si₂: The Hidden Order Mystery









 $\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$





 $\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$





 $\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$

=0.14 x 17.5 K =2.45 J/mol/K =0.42 R ln 2





 $\Delta S = \int_{0}^{T_{0}} \frac{C_{V}}{T} dT = 0.14 \text{ x } 17.5 \text{ K}$ =2.45 J/mol/K

=0.42 R ln 2

Large entanglement entropy.





 $\Delta S = \int_{0}^{T_{0}} \frac{C_{V}}{T} dT = 0.14 \text{ x } 17.5 \text{ K} \\ = 2.45 \text{ J/mol/K}$

=0.42 R ln 2

Large entanglement entropy.



What is the nature of the hidden order?















(NMR,MuSR).





(NMR,MuSR).



25 Years of Theoretical Proposals

25 Years of Theoretical Proposals

Landau Theory Shah et al. ('00) "Hidden Order"

Landau Theory

Itinerant

25 Years of Theoretical Proposals

Shah et al. ('00) "Hidden Order"
Ramirez et al, '92 (Quadrupolar SDW)
Ikeda and Ohashi '98 (d-density wave)
Okuno and Miyake '98 (composite)
Tripathi, Chandra, PC and Mydosh, '02 (orbital afm)
Dori and Maki, '03 (Unconventional SDW)
Mineev and Zhitomirsky, '04 (SDW)
Varma and Zhu, '05 (Spin-nematic)
Ezgar et al '06 (Dynamic symmetry breaking)
Fujimoto, '11 (Spin-nematic)
Ikeda et al '12 (Rank 5 nematic)
Rau and Kee '12 (Rank 5 pseudo-spin vector)
Tanmoy Das '12 (Topological Spin-nematic)

Itinerant

Local

25 Years of Theoretical Proposals

Landau Theory Shah et al. ('00) "Hidden Order" Ramirez et al, '92 (Quadrupolar SDW) Ikeda and Ohashi '98 (d-density wave) Okuno and Miyake '98 (composite) Tripathi, Chandra, PC and Mydosh, '02 (orbital afm) Dori and Maki, '03 (Unconventional SDW) Mineev and Zhitomirsky, '04 (SDW) Varma and Zhu, '05 (Spin-nematic) Ezgar et al '06 (Dynamic symmetry breaking) Fujimoto, '11 (Spin-nematic) Ikeda et al '12 (Rank 5 nematic) Rau and Kee '12 (Rank 5 pseudo-spin vector) Tanmoy Das '12 (Topological Spin-nematic) Barzykin & Gorkov, '93 (three-spin correlation) Santini & Amoretti, '94, Santini ('98) (Quadrupole order) Amitsuka & Sakihabara (Γ_5 , Quadrupolar doublet, '94) Kasuya, '97 (U dimerization) Kiss and Fazekas '04, (Rank 3 octupolar order) Haule and Kotliar '09 (Rank 4 hexa-decapolar) Rau and Kee '12 (Rank 5 pseudo-spin vector)

25 Years of Theoretical Proposals

Landau Theory Shah et al. ('00) "Hidden Order" Itinerant Ramirez et al, '92 (Quadrupolar SDW) Ikeda and Ohashi '98 (d-density wave) Okuno and Miyake '98 (composite) Tripathi, Chandra, PC and Mydosh, '02 (orbital afm) Dori and Maki, '03 (Unconventional SDW) Mineev and Zhitomirsky, '04 (SDW) Varma and Zhu, '05 (Spin-nematic) Ezgar et al '06 (Dynamic symmetry breaking) Fujimoto, '11 (Spin-nematic) Ikeda et al '12 (Rank 5 nematic) Rau and Kee '12 (Rank 5 pseudo-spin vector) Tanmoy Das '12 (Topological Spin-nematic) Barzykin & Gorkov, '93 (three-spin correlation) Local Santini & Amoretti, '94, Santini ('98) (Quadrupole order) Amitsuka & Sakihabara (Γ_5 , Quadrupolar doublet, '94) Kasuya, '97 (U dimerization) Kiss and Fazekas '04, (Rank 3 octupolar order) Haule and Kotliar '09 (Rank 4 hexa-decapolar) Rau and Kee '12 (Rank 5 pseudo-spin vector) Pepin et al '10 (Spin liquid/Kondo Lattice) Dubi and Balatsky, '10 (Hybridization density wave) Kondo Lattice

The Giant Ising Anisotropy.





$$M \propto \cos \left[2\pi \frac{\text{Zeeman}}{\text{cyclotron}} \right]$$













M. M. Altarawneh, N. Harrison, S. E. Sebastian, et al., PRL (2011). H. Ohkuni *et al.*, Phil. Mag. B 79, 1045 (1999).

16 spin zeros!

$$\frac{m^*}{m_e}g(\theta) = 2n+1$$



M. M. Altarawneh, N. Harrison, S. E. Sebastian, et al., PRL (2011). H. Ohkuni *et al.*, Phil. Mag. B 79, 1045 (1999).

16 spin zeros!

$$\frac{m^*}{m_e}g(\theta) = 2n+1$$

Quasiparticle with giant Ising anisotropy > 30. Pauli susceptibility anisotropy > 900



M. M. Altarawneh, N. Harrison, S. E. Sebastian, et al., PRL (2011). H. Ohkuni *et al.*, Phil. Mag. B 79, 1045 (1999).

16 spin zeros!

$$\frac{m^*}{m_e}g(\theta) = 2n+1$$

Quasiparticle with giant Ising anisotropy > 30. Pauli susceptibility anisotropy > 900

Superconductivity: Giant Ising Anisotropy

Quasiparticle with giant Ising anisotropy > 30. Pauli susceptibility anisotropy > 900

Superconductivity: Giant Ising Anisotropy



Quasiparticle with giant Ising anisotropy > 30. Pauli susceptibility anisotropy > 900

Superconductivity: Giant Ising Anisotropy



 $\mu_0 H_{\rm p} = \frac{2\Delta}{\sqrt{2} \ \mu_{\rm B} g_{\rm eff}^*}$

Quasiparticle with giant Ising anisotropy > 30. Pauli susceptibility anisotropy > 900


Quasiparticle with giant Ising anisotropy > 30. Pauli susceptibility anisotropy > 900

 $\langle \mathbf{k}\sigma | J_{\pm} | \mathbf{k}\sigma' \rangle = 0$



Ising QP's pair condense.

Quasiparticle with giant Ising anisotropy > 30. Pauli susceptibility anisotropy > 900

 $\langle \mathbf{k}\sigma | J_{\pm} | \mathbf{k}\sigma' \rangle = 0$





Ising QP's pair condense.

Quasiparticle with giant Ising anisotropy > 30. Pauli susceptibility anisotropy > 900

$$\langle \mathbf{k}\sigma | J_{\pm} | \mathbf{k}\sigma' \rangle = 0$$





Ising QP's pair condense.

Quasiparticle with giant Ising anisotropy > 30. Pauli susceptibility anisotropy > 900

Electrons hybridize with Ising 5f state to form Landau quasiparticles. $\langle {\bf k}\sigma|J_{\pm}|{\bf k}\sigma'
angle=0$

Hasta: Spear (Latin). A new kind of spinor order.

Rebecca FlintPremi Chandra(MIT)(Rutgers)

P.Chandra, R. Flint and P.C arXiv:1404.5920, *PRB* 86, 155155 (2012). doi:10.1038/nature11820 Piers Coleman Center for Materials Theory Rutgers, USA



Wills Lab, Bristol U 19 Jun 2014





Hasta: Spear (Latin). A new kind of spinor order.



Rebecca FlintPremi Chandra(MIT)(Rutgers)

P.Chandra, R. Flint and P.C arXiv:1404.5920, *PRB* 86, 155155 (2012). doi:10.1038/nature11820 Piers Coleman Center for Materials Theory Rutgers, USA



Wills Lab, Bristol U 19 Jun 2014

Take-home: Observation of (Perfect) Ising Quasiparticles





Altarawneh et al., PRL 108, 066407 (2012)

Hasta: Spear (Latin). A new kind of spinor order.

Piers Coleman Center for Materials Theory Rutgers, USA



Rebecca FlintPremi Chandra(MIT)(Rutgers)

P.Chandra, R. Flint and P.C arXiv:1404.5920, *PRB* 86, 155155 (2012). doi:10.1038/nature11820



Royal Holloway

University of London





Hasta: Spear (Latin). A new kind of spinor order.

Piers Coleman Center for Materials Theory Rutgers, USA



Rebecca FlintPremi Chandra(MIT)(Rutgers)

P.Chandra, R. Flint and P.C arXiv:1404.5920, *PRB* 86, 155155 (2012). doi:10.1038/nature11820



Wills Lab, Bristol U 19 Jun 2014



Hasta: Spear (Latin). A new kind of spinor order.

P.Chandra, R. Flint and P.C

Wills Lab, Bristol U 19 Jun 2014

90° Take-home: Observation of (Perfect) Ising Quasiparticles Many body hybridization Spin 1/2 e \rightleftharpoons Integer spin (J_z=±1) Doublets [100] Proposal: Order parameter is a spinor (mixing J & J+1/2, breaking <u>double</u> time reversal) [001] $\Theta^2 = (-1)^{2J}$ Altarawneh et al., PRL 108, 066407 (2012)

arXiv:1404.5920, PRB 86, 155155 (2012).

doi:10.1038/nature11820

Rebecca Flint Premi Chandra (MIT) (Rutgers)





Piers Coleman



















doi:10.1038/nature11820 Hastatic order in the heavy-fermion compound URu_2Si_2

Premala Chandra¹, Piers Coleman^{1,2} & Rebecca Flint³

Ψ

Order parameter carries half-integer spin







doi:10.1038/nature11820 Hastatic order in the heavy-fermion compound URu₂Si₂

Premala Chandra¹, Piers Coleman^{1,2} & Rebecca Flint³

Hasta: Spear (Latin)

 Ψ

Order parameter carries half-integer spin

"Spinor"

 $\Psi(R\mathbf{k}) = D(R)\Psi(\mathbf{k})$

a)





In conventional heavy fermion materials a hybridization derives from virtual excitations between a Kramers doublet and an excited singlet.





In conventional heavy fermion materials a hybridization derives from virtual excitations between a Kramers doublet and an excited singlet. A uniform hybridization breaks no symmetry and develops as a cross-over.



H. Amitsuka and T. Sakakibara, J. Phys. Soc. Japan 63, 736-47 (1994).

But if the ground-state is a non-Kramer's doublet, the Kondo effect occurs via an *excited Kramer's doublet*.



non-
Kramers
$$\Gamma_5$$
 ===== $|5f^2, \alpha\rangle = \hat{\chi}^{\dagger}_{\alpha}|0\rangle$ (K=+1)

H. Amitsuka and T. Sakakibara, J. Phys. Soc. Japan 63, 736-47 (1994).

But if the ground-state is a non-Kramer's doublet, the Kondo effect occurs via an *excited Kramer's doublet*.



Kramers
(K=-1)
$$\Gamma_7 = [5f^3, \sigma\rangle = \hat{\Psi}^{\dagger}_{\sigma}|0\rangle$$

non-
Kramers
(K=+1) $\Gamma_5 = [5f^2, \alpha\rangle = \hat{\chi}^{\dagger}_{\alpha}|0\rangle$

H. Amitsuka and T. Sakakibara, J. Phys. Soc. Japan 63, 736-47 (1994).

But if the ground-state is a non-Kramer's doublet, the Kondo effect occurs via an *excited Kramer's doublet*.





H. Amitsuka and T. Sakakibara, J. Phys. Soc. Japan 63, 736-47 (1994).

But if the ground-state is a non-Kramer's doublet, the Kondo effect occurs via an *excited Kramer's doublet*.





H. Amitsuka and T. Sakakibara, J. Phys. Soc. Japan 63, 736-47 (1994).

But if the ground-state is a non-Kramer's doublet, the Kondo effect occurs via an *excited Kramer's doublet*.







H. Amitsuka and T. Sakakibara, J. Phys. Soc. Japan 63, 736-47 (1994).

But if the ground-state is a non-Kramer's doublet, the Kondo effect occurs via an *excited Kramer's doublet*.





 $|5f^3,\sigma\rangle\langle 5f^2,\alpha| = \hat{\Psi}^{\dagger}_{\sigma}\hat{\chi}_{\alpha}$

H. Amitsuka and T. Sakakibara, J. Phys. Soc. Japan 63, 736-47 (1994).

But if the ground-state is a non-Kramer's doublet, the Kondo effect occurs via an *excited Kramer's doublet*.





 $|5f^3,\sigma\rangle\langle 5f^2,\alpha| = \hat{\Psi}^{\dagger}_{\sigma}\hat{\chi}_{\alpha}$

"Hastatic" order.





$$5f^3, \sigma \rangle \langle 5f^2, \alpha | = \hat{\Psi}^{\dagger}_{\sigma} \hat{\chi}_{\alpha}$$

"Hastatic" order.





$$|5f^3,\sigma\rangle\langle 5f^2,\alpha|\longrightarrow \langle\hat{\Psi}^{\dagger}_{\sigma}\rangle\hat{\chi}_{\alpha}$$



("Magnetic Higgs Boson")

$$\Psi = \begin{pmatrix} \langle \Psi_{\uparrow} \rangle \\ \langle \Psi_{\downarrow} \rangle \end{pmatrix}$$

$$|5f^3,\sigma\rangle\langle 5f^2,\alpha|\longrightarrow \langle\hat{\Psi}^{\dagger}_{\sigma}\rangle\hat{\chi}_{\alpha}$$



("Magnetic Higgs Boson")

$$\Psi = \begin{pmatrix} \langle \Psi_{\uparrow} \rangle \\ \langle \Psi_{\downarrow} \rangle \end{pmatrix}$$

$$|5f^3,\sigma\rangle\langle 5f^2,\alpha|\longrightarrow \langle\hat{\Psi}^{\dagger}_{\sigma}\rangle\hat{\chi}_{\alpha}$$

Quasiparticles acquire the Ising anisotropy of the non-Kramers doublet.



Open Challenges.

- QCPs: Origin of C/T ~ $Log(T_0/T)$? ρ ~ T?
- Co-existence heavy fermions & LM AFM = Two fluid behavior? [Supersymmetry? B/F]
- HFSC: how is the spin incorporated into the condensate? [Composite pairs?]
- Hidden order (HO). Origin of ISING qps?
 [1/2 integer spinor OP]
- HO: complex MDW (multipolar density wave) vs Fractional spinor order.

Thank You!